THE DISCHARGE LIQUID FLOW FROM THE TURBINE IMPELLER* **

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The flow is described of a stream of liquid ejected by the blades of a rotating turbine impeller in a cylindrical vessel with flat bottom and radial baffles under the turbulent regime of the mixed charge. The flow is described on one hand as a field of streamlines in the examined stream, and, on the other hand, in terms of the volumetric flow rate in various cross-sections of the stream.

In the mixed system with a turbine impeller and radial baffles consider a cylindrical volume of radius (radial coordinate) r and height (axial coordinate) z defined as a vertical distance between two closest places z_1 and z_2 in the mixed system where the radial component of local mean velocity $\bar{w}_{rad}$ reaches zero. The origin of the frame of reference is put in the point of intersection of the axis of symmetry of the vessel with the horizontal plane of symmetry of the impeller. The axial coordinate of this plane is taken equal zero. The mixed system is regarded as axially symmetric and the mixed batch as incompressible. Each point in the defined region is assigned its mean velocity vector and the family of tangential curves to these vectors forms the field of streamlines. For axially symmetric systems one has[1]

$$\bar{w}_{rad}(r, z) = (1/r)\, \partial\psi(r, z)/\partial z \qquad (1)$$

from which

$$\psi(r, z) = \int_{z_{max}}^{z} r\bar{w}_{rad}(r, z)\, dz\,. \qquad (2)$$

In accord with the form of the velocity profile, $\bar{w}_{rad} = \bar{w}_{rad}(z)$, in the stream exiting from the blades of the turbine impeller the profile of the dimensionless radial velocity component is taken as

$$W_{rad}(r, Z) = \bar{w}_{rad}(r, Z)/\pi\, dn = A(r) + B(r)\, Z + C(r)\, Z^2\, dZ\,. \qquad (3)$$

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The constants A, B and C of the profile $W_{rad} = W_{rad}(Z)$, (r = const.), may be estimated from the experimental profile by statistical methods. The dimensionless stream function is obtained from Eqs (*2*) and (*3*):

$$\Psi(r, Z) = \psi(r, Z)/nd^3 = (\pi h/2d^2) \int_{Z_{max}}^{Z} r[A(r) + B(r)\, Z + C(r)\, Z^2]\, dZ\,. \qquad (4)$$

For the lower integration limit we take the dimensionless axial coordinate, Z_{max}, of the point where the radial velocity component reaches a maximum. The maximum value of W_{rad} along the $W_{rad} = W_{rad}(Z)$, (r = const.), curve may be calculated as the maximum of the function in Eq. (*3*). Then

$$(W_{rad})_{max} = A(r) - B^2(r)/4C(r)\,, \quad (r = \text{const.})\,. \qquad (5)$$

For the coordinates Z_1 and Z_2 of the points delimiting the cylindrical region in the vertical direction one must have that $W_{rad}(r, Z) = 0$ (Eq. (*3*)), so that the values of the above given coordinates are

$$Z_{1,2}(r) = \{-B(r) \pm [B^2(r) - 4A(r)\, C(r)]^{1/2}\}/2C(r)\,, \quad (r = \text{const.})\,. \qquad (6)$$

On integrating the profile of the radial component over the whole width of the examined region one obtains the total flow rate on the given radius r. In the dimensionless form the flow rate is given as

$$K_t(r) = (V_t(r)/nd^3 = 2\pi\{2[\Psi_{max}(r) - \Psi(Z_{max})]\} =$$
$$= (\pi^2 h)/d^2 \int_{Z_1}^{Z_2} r[A(r) + B(r)\, Z + C(r)\, Z^2]\, dZ =$$
$$= (\pi^2 r h)/d^2[A(r)\,(Z^2 - Z_1) + 1/2B(r)\,(Z_2^2 - Z_1^2) + 1/3C(r)\,(Z_2^3 - Z_1^3)]\,,$$
$$(r = \text{const.})\,. \qquad (7)$$

For $r = d/2$ we speak of the so called pumping capacity of the impeller, *i.e.* a volume of liquid ejected from the space occupied by the blades per unit time. Let us note still that the quantity $\Psi_{max}(r)$ represents the value of the stream function at Z_1 and Z_2; $\Psi(Z_{max})$ then the value of the stream function at Z_{max} on the $W_{rad} = W_{rad}(Z)$ profile and a given r.

EXPERIMENTAL

The experiments were carried out in a cylindrical vessel with flat bottom 1 000 mm in inner diameter equipped with four radial baffles $b = 0{\cdot}1\ D$ wide mounted on the wall. The measurements were carried out with distilled water filled in the vessel to a height H of the liquid at rest, H being equal the inner diameter D. A six-flat-blade turbine impeller, its size being $d/D = 1/3$ or $1/4$, was located in the vessel coaxially its lower edge reaching $h_2 = 0.3\ D$ above the bottom of the vessel. A five-hole Pitot tube was used to measure local velocity profiles in the stream of liquid between the outer edge of the rotating blades and the wall of the vessel. The Pitot tube enabled the magnitude and the direction of the mean velocity vector to be determined in a given position of the examined stream[1,2].

RESULTS AND DISCUSSION

The experimental results of measurements of the total pressure by the five-hole Pitot tube served to determine individual components of the local mean velocity vector as well as the angles (Fig. 1) of the velocity vector made with its projection $\mathbf{w}_k$ (angle β) and the angle made by the unit vector $\mathbf{e}_r$ with the rectangular projection of the velocity vector $\mathbf{w}_k$ (angle α). The method of evaluation has been described earlier[2,7]. The found profiles of $W_{rad} = W_{rad}(Z)$, (r = const.) were used to determine the constants A, B, C of Eq. (3) by the least square method which in turn served to determine the distribution $\Psi = \Psi(Z)$ in a given radial position by numerical integration. Eq. (7) was then used to compute the value of $K_t(r)$ – the flow criterion. The results for several number of the impeller revolution were then averaged. The profiles $\Psi = \Psi(r, Z)$

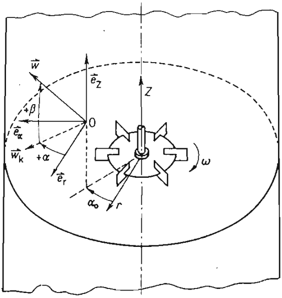

FIG. 1

Orientation of the Velocity Vector in the Mixed System

Vectors indicated by arrows.

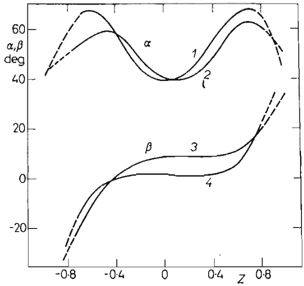

FIG. 2

Axial Profiles of α and β in the Stream Leaving the Blades of the Turbine Impeller

1 $\alpha = \alpha(Z)$, $d/D = 1/4$, *2* $\alpha = \alpha(Z)$, $d/D = 1/3$, *3* $\beta = \beta(Z)$, $d/D = 1/3$, *4* $\beta = \beta(Z)$, $d/D = 1/4$.

were then evaluated from the found profiles $\Psi = \Psi(Z)$, (r = const.) by graphical interpolation.

Profiles of the angle α. All profiles of α are of similar form and independent of the frequency of the impeller. The function $\alpha = \alpha(Z)$ reaches a minimum in the neighbourhood of the horizontal plane of symmetry of the impeller (Fig. 2), but somewhat shifted toward its upper edge. In the core of the stream the radial direction of the flow prevails. The surface layers of the stream (near the edges of the blades) display visible straightening of the tangential direction of the flow deccelerated by the surrounding liquid; in the bottom part the effect is combined with that of the bottom.

Unlike the results of Cooper and Wolf[3] the angle α does not reach minimum at $Z = \pm 1$. With increasing radial distance α decreases. Assuming two-dimensional flow and ideal liquid the decrease may be explained by the following reasoning: Denoting by $\alpha|_r$ the angle of the stream in a certain radial distance we have that (Fig. 3)

$$\sin \alpha|_r = (d/2r) \sin \alpha|_{r=d/2}\,, \quad [r \geqq d/2]\,. \tag{8}$$

From the known profile of the angle $\alpha|_{r=d/2}$ in the immediate vicinity of the impeller one can estimate the profile in a greater radial distance r from the axis.

Profiles of the angle β *and the dimensionless axial component of mean velocity.* Our experiments show the angle β and hence the axial component of mean velocity to have a greater importance than that given them thus far[3-5]. In the core of the stream (near $Z = 0$) the values of β may be small but in the surface layers of the stream β reaches as much as 10° to 20°. From the graph in Fig. 2 it is apparent that starting from approximately $Z = -0.4$ β takes positive values indicating that the flow within the stream is not symmetric with its greater portion being diverted upwards toward the free level of the charge. Simultaneously, one can detect apparent dependence on the d/D ratio. The value of the axial component of the mean velocity reaches in the core roughly 25%, in the surface layer of the stream ($Z = 0.8$) as much as 50% of the radial component. Clearly, in a vessel with a larger impeller relatively to the vessel diameter the effect of the radial baffles and the wall will become more manifest.

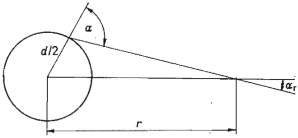

FIG. 3
Angle α as a Function of Radial Distance

This is illustrated also in Fig. 2. The curves indicating the course of the angle β are markedly different suggesting that the axial component of the velocity is not negligible. All velocity profiles are clearly asymmetric – the stream leaving the blades of the impeller is from the very beginning diverted toward the free level of the charge. As another effect combining with the previous one is that the path along the wall and the bottom is shorter and hence the losses in the lower part of the vessel are larger than in the upper one.

Profiles of the dimensionless radial component of mean velocity. As an example may serve the values of the dimensionless radial velocity plotted in Fig. 4 for several frequencies of the impeller. The figure indicates clearly that the plotted quantity is independent of the number of the impeller revolution as had been anticipated; all measurements were carried out under the turbulent automodel regime of the flow, *i.e.* the Reynolds number exceeding $9 \cdot 10^4$.

Eq. (*3*) was used as a fitting equation the constants of which were determined by the least square method from the known values of the radial velocity component. The constants are summarized in Table I and solid lines in Figs 4 and 5 indicate the fitted curves.

The derived form of the fitting equation suits well in most of the examined velocity profiles. The radial velocity component reaches a maximum around the horizontal plane of symmetry of the impeller in its immediate vicinity. Toward the outer edges of the impeller it falls rapidly to reach a minimum already at $Z \approx \pm 0{\cdot}75$. This suggests that the stream ejected from the blades of the impeller is narrower than the width of the blades (Fig. 5). The values of the axial shift of the velocity profiles in the immediate vicinity of the impeller are relatively small but grow with radial distance from the impeller and, moreover, display a marked dependence on the d/D ratio.

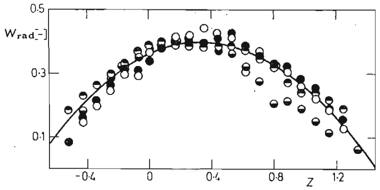

FIG. 4

Axial Profile of the Dimensionless Radial Component of Mean Velocity in the Stream between Rotating Impeller and Wall ($d/D = 1/3$, $r = 0{\cdot}25$ m)

n: ● 50 min^{-1}, ○ 75, ◓ 100, ◒ 118.

TABLE I

Parameters of the Velocity Profile $W_{rad} = W_{rad}(Z)$ in the Stream Leaving the Blades of the Turbine Impeller

$d/D = 1/4$					$d/D = 1/3$				
r m	A	B	C	K_t	r m	A	B	C	K_t
0·128[a]	0·669	0·026	−1·111	0·70	0·169[a]	0·640	0·210	−1·068	0·76
0·140	0·676	0·024	−1·488	0·67	0·187	0·643	0·261	−1·498	0·72
0·155	0·690	0·093	−1·484	0·77	0·207	0·486	0·340	−1·107	0·74
0·170	0·574	0·204	−0·922	0·83	0·250	0·354	0·226	−0·332	0·84
0·187	0·465	0·062	−0·314	1·12					

[a] Pumping capacity of the turbine impeller.

The field of streamlines leaving from blades of the turbine impeller. The field of streamlines may be determined from the known velocity profiles. For such a purpose, however, one has to define the value of the stream function on the limits of the examined system, or a reference streamline. The probe used was calibrated by velocities exceeding 0·25 m/s, and, accordingly, the values found in the system below this limit are not quite reliable. In addition, the fitting equation does not follow very well the low velocity pattern in the outer layers of the stream ejected from the blades

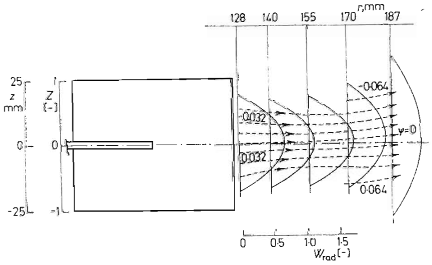

FIG. 5

Axial Profiles of the Dimensionless Radial Component of Mean Velocity and the Field of Streamlines in the Stream between Impeller and Wall ($d/D = 1/4$)

The figure on each streamline indicates the assigned value of the stream function.

of the impeller. As a reference value we thus took the streamlines originating on a given radius at the point Z_{max} where the stream function was assigned a zero value. All other streamlines were related to this streamline. The streamlines, as envelopes of the time-averaged velocity vectors, thus represent an averaged picture of constantly changing reality. The streamlines in Fig. 5 indicate clearly the asymmetric character of the stream ejected from the blades of the turbine impeller already mentioned above.

From the papers that appeared in the literature to date the stream patters were published only by De Souza and Pike[4] and Nagata and coworkers[6]. The former authors present a theoretical picture of the streamline field obtained on the basis of a model; the latter authors did not indicate the method leading to their flow pattern. Moreover, their geometrical arrangement is entirely different and thus no comparison can be afforded.

The total flow rate in the stream of liquid leaving the blades of the turbine impeller depends both on the relative size of the impeller and the radial distance from the impeller. Within a certain range of r the value K_t is almost constant; the flow rate through the impeller is not markedly affected by the liquid entrained from the vicinity of the stream. Outside this region the entrainment becomes significant (Fig. 5) and the value of K_t increases. It may be concluded that the mentioned increase induced by the change of the tangential component into the radial one due to the action of the baffles could be detected even in regions closer to the walls which were not examined in this work. This seems to be confirmed also by the streamline pattern of Nagata and coworkers[6].

LIST OF SYMBOLS

$A(r)$	coefficient in Eq. (*3*)
b	width of radial baffle (m)
$B(r)$	coefficient in Eq. (*3*)
$C(r)$	coefficient in Eq. (*3*)
d	diameter of impeller (m)
$\mathbf{e}_r$	unit vector
D	diameter of vessel (m)
h	height of impeller blade (m)
h_2	height of lower edge of blade above bottom (m)
H	liquid height at rest (m)
K_t	dimensionless flow rate
n	frequency of impeller revolution (s^{-1})
r	radial coordinate (m)
R	dimensionless radial coordinate
V_t	total flow rate of mixed charge ($m^3 s^{-1}$)
$\mathbf{w}_k$	projection of velocity vector ($m s^{-1}$)
w_{rad}	radial component of local mean velocity ($m s^{-1}$)
W_{rad}	dimensionless radial component of local mean velocity

z axial coordinate (m)
$Z = 2z/h$ dimensionless axial coordinate
α angle made by vector of local mean velocity and its radial projection (deg)
β angle made by vector of local mean velocity and its axial projection (deg)
ψ stream function ($m^3 s^{-1}$)
Ψ dimensionless stream function

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